Problem 3

Solve the equation $|2x-1|-|x+5|=3$.

Solution

Isolate one of the terms with an absolute value sign.

$$|2x-1| = 3+|x+5| \tag{1}$$

Remove the absolute value sign on the left by placing $\pm$ on the right side.

$$2x-1 = \pm\left(3+|x+5|\right)$$

As a result, equation (1) has split into two.

$$2x-1 = 3+|x+5| \quad \text{or} \quad 2x-1 = -3-|x+5|$$

Isolate the remaining term with an absolute value sign.

$$|x+5| = 2x-4 \quad \text{or} \quad |x+5| = -2x-2 \tag{2}$$

Remove the absolute value sign in each equation by placing $\pm$ on the right side.

$$x+5 = \pm(2x-4) \quad \text{or} \quad x+5 = \pm(-2x-2)$$

As a result, each of these equations in (2) has split into two.

$$x+5 = 2x-4 \quad \text{or} \quad x+5 = -2x+4 \quad \text{or} \quad x+5 = -2x-2 \quad \text{or} \quad x+5 = 2x+2$$

Solve each of these equations for x.

$$x = 9 \quad \text{or} \quad x = -\frac{1}{3} \quad \text{or} \quad x = -\frac{7}{3} \quad \text{or} \quad x = 3$$

Now, one by one, check to see whether these values of x satisfy the original equation.

$$x = 9: \qquad |2x-1|-|x+5| = |2(9)-1|-|(9)+5| = 17-14 = 3$$

$$x = -\frac{1}{3}: \qquad |2x-1|-|x+5| = \left|2\left(-\frac{1}{3}\right)-1\right|-\left|\left(-\frac{1}{3}\right)+5\right| = \frac{5}{3}-\frac{14}{3} = -3$$

$$x = -\frac{7}{3}: \qquad |2x-1|-|x+5| = \left|2\left(-\frac{7}{3}\right)-1\right|-\left|\left(-\frac{7}{3}\right)+5\right| = \frac{17}{3}-\frac{8}{3} = 3$$

$$x = 3: \qquad |2x-1|-|x+5| = |2(3)-1|-|(3)+5| = 5-8 = -3$$

Therefore,

$$x = \left\{-\frac{7}{3}, 9\right\}.$$

The graph below verifies that these are in fact the only two solutions.